

Sign Change Fault Attacks On Elliptic Curve Cryptosystems

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Content

Fault Model

- Sign Change Faults

Attack

- on Elliptic Curve Scalar Multiplication

Countermeasure

- against Sign Change Attacks

Elliptic Curves: Notation

- Set of points $(x : y : z)$ satisfying

$$\Rightarrow E_p : y^2 z \equiv x^3 + Axz^2 + Bz^3 \pmod{p} \quad (1)$$

(Weierstraß-Equation in projective coordinates)

- We only consider E defined over \mathbb{F}_p , p prime
- Group of points: All points satisfying (1)
 - $(x : y : z) = (\lambda x : \lambda y : \lambda z)$ for $\lambda \neq 0$
 - $\mathcal{O} = (0 : 1 : 0) \longrightarrow$ “point at infinity”

Previous Work

Approaches:

- Analysis of faults in curve and field parameters (Biehl, Meyer, Müller 2000 & Ciet, Joye 2003)

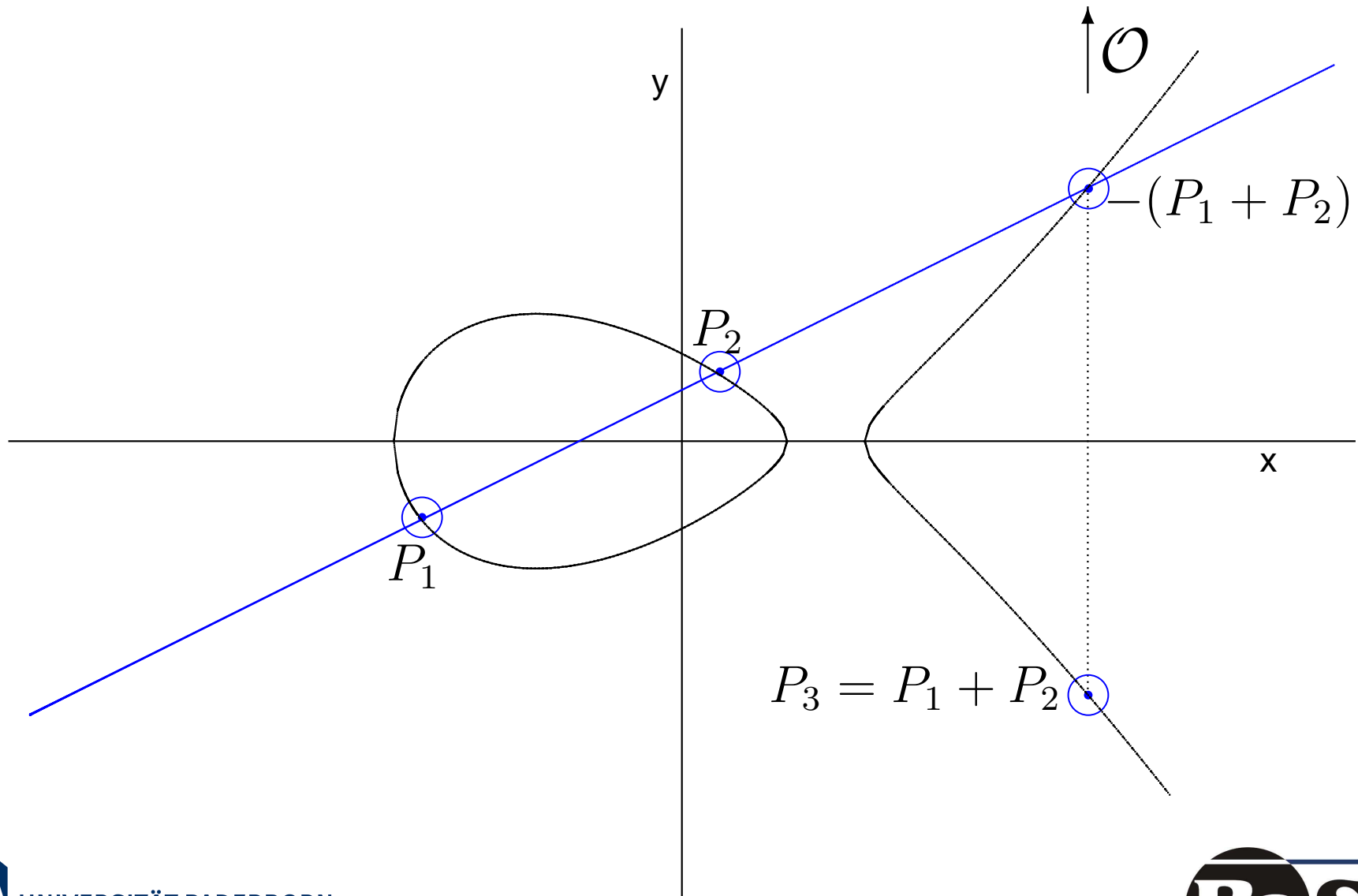
Results:

- With overwhelming probability, a (random) fault results in a final result that is not on the curve.

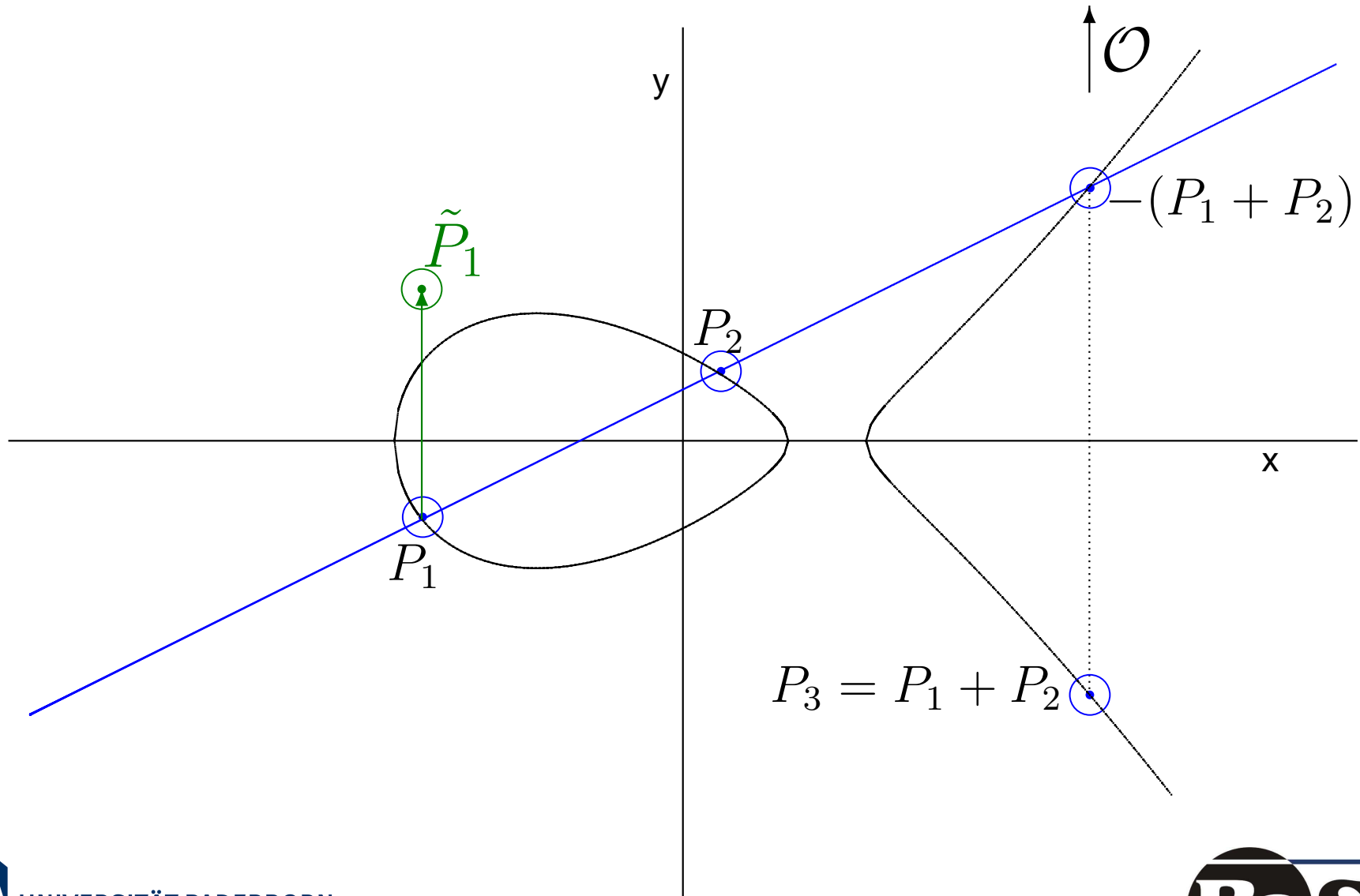
Natural Countermeasure:

- Check result before output: Is result a valid point on curve?

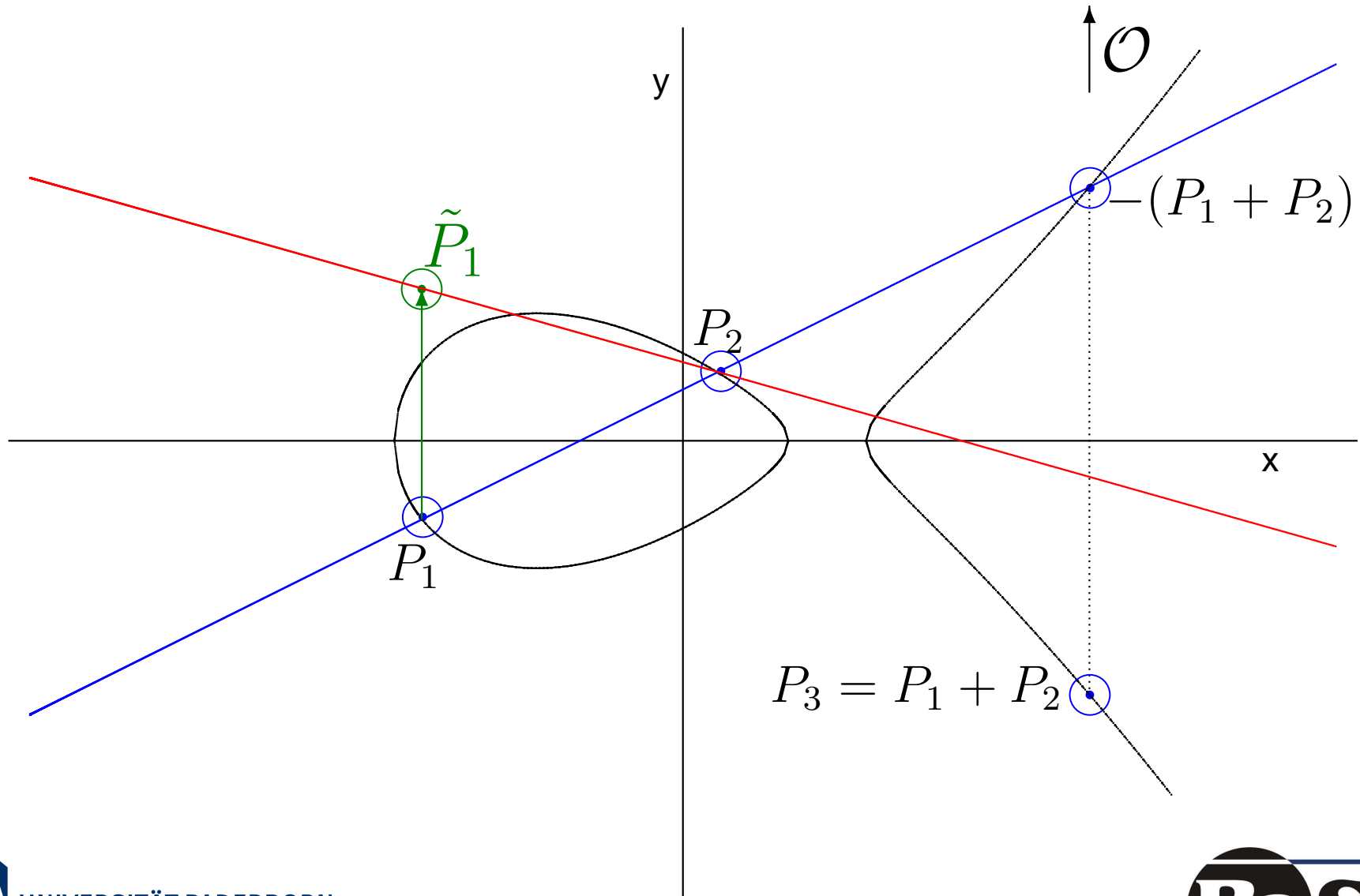
Fault Attacks on Elliptic Curve Addition



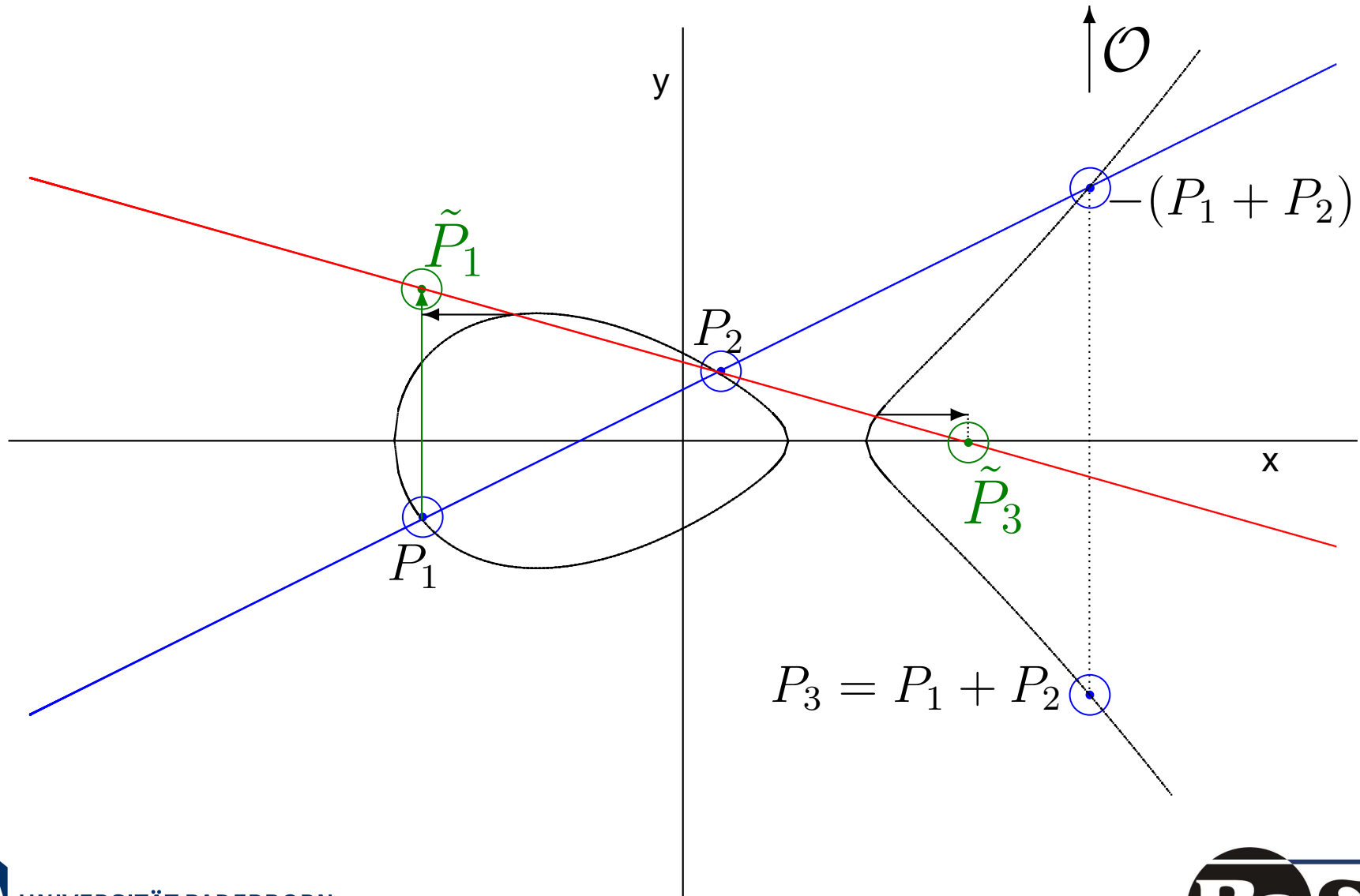
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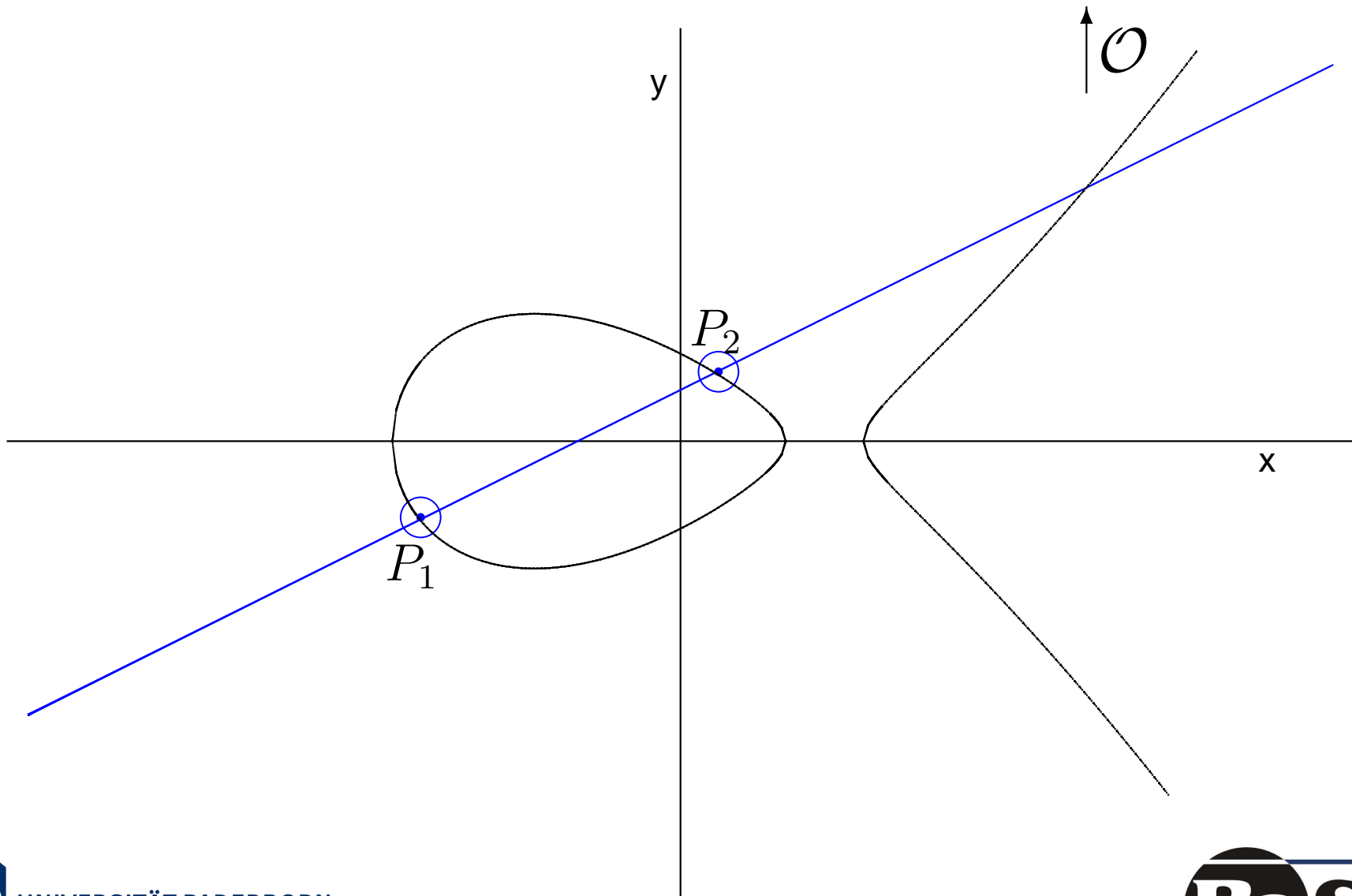
Fault Attacks on Elliptic Curve Addition



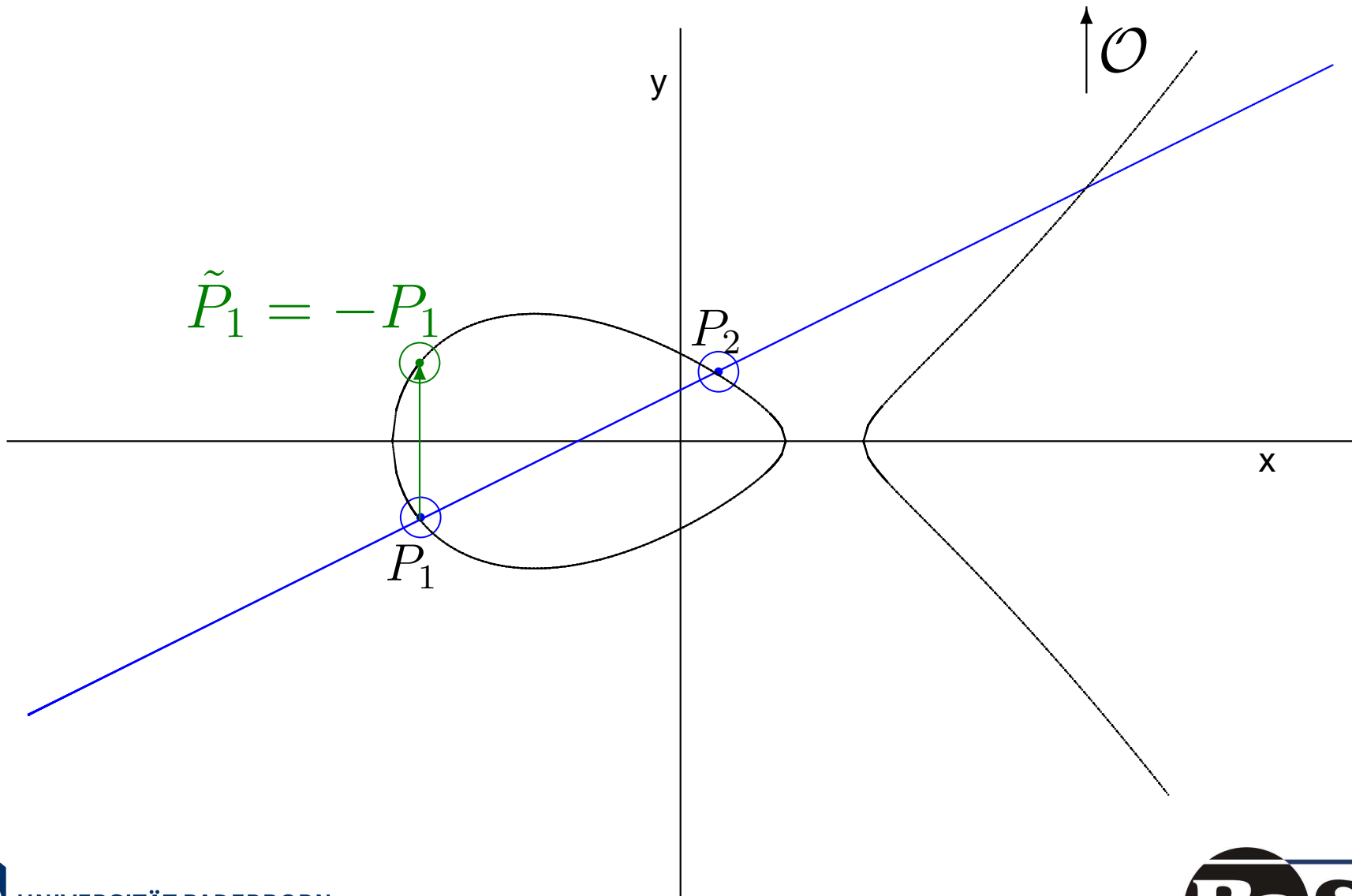
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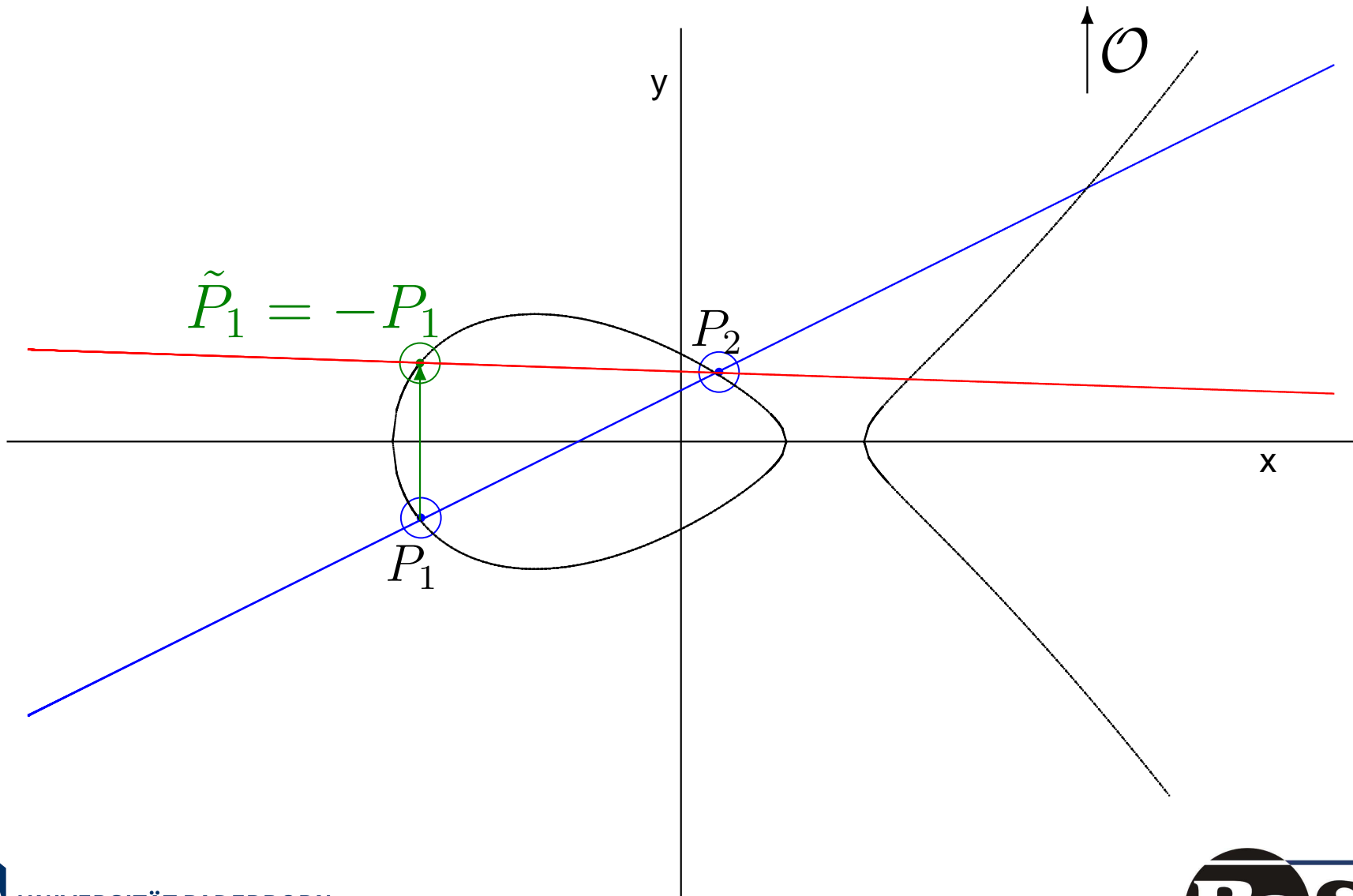
Sign Change Faults



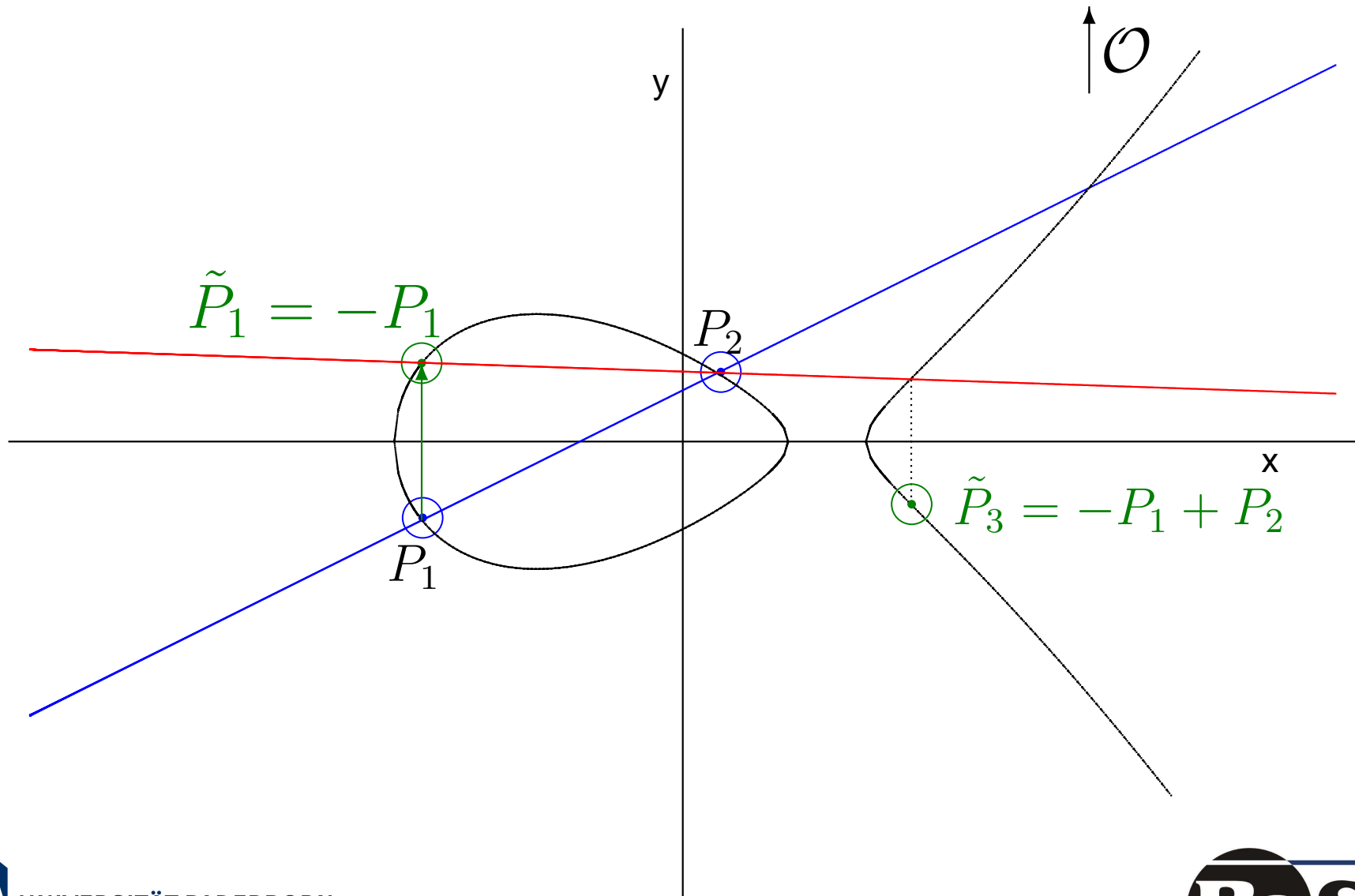
Sign Change Faults



Sign Change Faults



Sign Change Faults



Can we achieve Sign Change Faults?

Yes, we can! Examples:

- NAF-based scalar multiplication: attack a key bit
- Attacks during point addition, e.g., affine addition:

$$\lambda = \frac{y_1 - y_2}{x_1 - x_2}, \quad x_3 = \lambda^2 - x_1 - x_2$$
$$y_3 = -y_1 + \lambda \cdot (x_1 - x_3)$$

- Change y_2 to achieve a Sign Change Fault
- Attack ALU s.t. argument is inverted
- Success depends on the implementation (hardware and software)

Fault Attacks With Sign Change Faults (1)

Compute $Q = kP$ on E_p :

0 Set $n := l(k)$

1 Set $Q_n := \mathcal{O}$

2 For i from $n-1$ to 0 do

3 Set $Q'_i := 2 \cdot Q_{i+1}$

4 If $(k_i = 1)$ then set $Q_i := Q'_i + P$
 else set $Q_i := Q'_i$

5 Return Q_0

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Attacking

$$Q'_i \xrightarrow{\text{fault}} -Q'_i$$

at random

iteration i

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Attacking

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Step 1: Describe faulty final result

$$\tilde{Q} = -Q + 2 \cdot L_i(k), \text{ where } L_i(k) := \sum_{j=0}^i k_j 2^j \cdot P$$

Fault Attacks With Sign Change Faults (2)

Step 2: Collect many faulty final results

- Choose block size $m \Rightarrow O(2^m)$ operations:
- Mount $(n/m) \log(2n)$ many attacks to hit every possible block with Prob. at least $1/2$

Fault Attacks With Sign Change Faults (2)

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Step 3: incremental computation of k

- Assumption: all s lowest bits of k are known
- try all possibilities with up to $s + m$ bits:

$$\tilde{Q} \stackrel{?}{=} -Q + 2 \cdot L_{s+m-1}(k)$$

- Compare to gathered faulty final results

Fault Attacks With Sign Change Faults (3)

Step 4: Proof of correctness

- guessed pattern describes a faulty final result \tilde{Q} :
we show: pattern correct
- if no pattern describes \tilde{Q} :
 - “Zero Block Failure“: k has block of zeros

Summary of Attack

Theorem: Secret scalar k of length n is recovered with

$$O(n \cdot 2^m \cdot t)$$

scalar multiplications with probability at least $1/2$ inducing $t = (n/m) \log(2n)$ Sign Change Faults.

- This attack also applies to other scalar multiplication algorithms (NAF-LR/RL, Montgomery Ladder)
- Attack layout derived from first fault attack on RSA (Boneh, DeMillo, Lipton 1997)

Countermeasure Against Sign Change Attacks

What we want

- check final result efficiently for correctness

Idea

- Check using a “small” curve:
Choose prime t and (A_t, x_t, y_t) to define

$$E_t : y^2 z \equiv x^3 + A_t x z^2 + B_t z^3 \pmod{t}$$

$$P_t = (x_t : y_t : 1)$$

such that order of E_t is prime

A “combined“ curve

- Determine $E_{pt} : y^2 z \equiv x^3 + A_{pt} x z^2 + B_{pt} z^3 \pmod{pt}$

$$P_{pt} = (x_{pt} : y_{pt} : 1)$$

- Requirement: $A_{pt} \equiv A \pmod{p}$

$$A_{pt} \equiv A_t \pmod{t} \text{ etc.}$$

- Using the Chinese Remainder Theorem (CRT):

$$A_{pt} := \text{CRT}(A, A_t) \text{ etc.}$$

- First, compute $Q_{pt} := kP_{pt}$ on E_{pt}

We have $Q_{pt} \equiv kP \pmod{p}$ and

$$Q_{pt} \equiv kP_t \pmod{t}$$

A New SCF-Secure Algorithm for $k \cdot P$

SCF-Secure Scalar Multiplication $Q = kP$

Precomputation (during production time of device)

1 Choose prime t and “small” curve E_t

2 Determine the “combined” curve E_{pt}

Main (computations on the device)

3 Set $Q := kP_{pt}$ on E_{pt}

4 Set $R := kP_t$ on E_t

5 If $R \not\equiv Q \pmod{t}$ then **output** „failure“ else **output** Q on E_p

● Analysis: Order of E_t is security parameter

● undetectable faults: Adversary needs $O(2^{\text{ord}(E_t)})$ guesses

Conclusion

Summary:

- New Sign Change Attacks
- New Countermeasure

Open Problems:

- Extend the idea to curves over binary fields
- Other specialized fault types?

Thank you!

Appendix

On Choosing E_t

Theorem 1: (Hasse)

Given $E_p : y^2 z \equiv x^3 + Axz^2 + Bz^3 \pmod{p}$, it is

$$p + 1 - 2\sqrt{p} \leq \#E_p \leq p + 1 + 2\sqrt{p}.$$

Fact 2: $\exists p^2 - p$ different elliptic curves over \mathbb{F}_p .

Theorem 6: (Deurich)

There exists a constant $c > 0$ such that there are at least $c \cdot (p\sqrt{p}) / \log(p)$ many elliptic curves for every given group order.

On Choosing E_t

Conjecture 7: (Cramer and Goldwasser/Kilian)

There exist constants $c_1, c_2 > 0$ such that

$$\pi(t + 2\sqrt{t}) - \pi(t - 2\sqrt{t}) \geq c_2 \sqrt{t} / \log^{c_1}(t).$$

Theorem 8: Choose $(A_t, x_t, y_t) \in \mathbb{Z}_t^3$ uniformly at random. (A_t, x_t, y_t) defines E_t uniquely. If Conjecture 7 is true, then $\exists c > 0$ such that the probability that E_t has prime order is at least

$$\frac{c \cdot c_2}{\log^{1+c_1}(t)},$$

where c_1, c_2 are as in Conjecture 7.

Montgomery's skalar Multiplikation

Montgomery Algorithm: $Q = k \cdot P$

init $P1_{(n-1)} := P$ and $P2_{(n-1)} := 2P$ and $n := bits(k)$

main for i from $n-2$ downto 0 do

if $(k_i = 0)$ then set $P2_{(i)} := P1_{(i+1)} + P2_{(i+1)}$

$P1_{(i)} := 2P1_{(i+1)}$

if $(k_i = 1)$ then set $P1_{(i)} := P1_{(i+1)} + P2_{(i+1)}$

$P2_{(i)} := 2P2_{(i+1)}$

output $Q = P1_{(0)}$

$$\tilde{Q} = \left(\frac{l_i(k)}{2^i} - 1 \right) \cdot (Q - l_i(k)P) + l_i(k)P, \text{ where } l_i(k) = \sum_{j=0}^i k_j 2^j$$

Projektive Addition

$$P_1 = (x_1 : y_1 : z_1), P_2 = (x_2 : y_2 : z_2), P_1 + P_2 = P_3 = (x_3 : y_3 : z_3)$$

$$x_3 := r^2 - tw^2$$

$$2y_3 := vr - mw^3$$

$$z_3 := z_1 z_2 w,$$

where

$$u_1 := x_1 z_2^2, \quad s_1 := y_1 z_2^3, \quad w := u_1 - u_2, \quad r := s_1 - s_2,$$

$$u_2 := x_2 z_1^2, \quad s_2 := y_2 z_1^3, \quad t := u_1 + u_2, \quad m := s_1 + s_2,$$

and $v := tw^2 - 2x_3$