Differential Fault Analysis on the SHA1 Compression Function

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FDTC 2011 - 28. 09. 2011

Citation bug

- Due to Bibtex-error please correct:

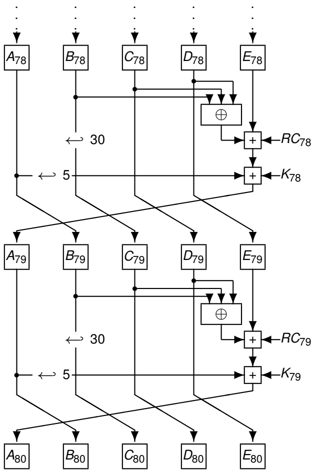
[20] Ruilin Li, Chao Li and Chunye Gong. Differential Fault Analysis on SHACAL-1. In *FDTC* 2009, pages 120-126. IEEE Computer Society, 2009.

Agenda

- Previous work: DFA on SHACAL1 (Ruilin Li et al., FDTC 2009)
- Extension to the SHA1 compression function
- DFA on SHA1COMPR
- Computational optimization
- Fault model
- Simulation results
- Conclusion and ongoing work

SHACAL1

- SHACAL1 is a symmertic 160-bit block cipher
- $\text{SHACAL1} : \{0,1\}^{160} \times \{0,1\}^{512} \longrightarrow \{0,1\}^{160},$
  $(P, K) \longmapsto C$
- It uses the building blocks of the SHA1 hash function without the final addition
- $P = A_0 || B_0 || C_0 || D_0 || E_0$
- $K = K_0 || K_1 || \ldots || K_{15}$ and
  $K_i = (K_{i-3} \oplus K_{i-8} \oplus K_{i-14} \oplus K_{i-16}) \hookleftarrow 1,$
  $i = 16, 17, \ldots, 79$
- $C = A_{80} || B_{80} || C_{80} || D_{80} || E_{80}$



DFA on SHACAL1 - Theoretical background

- SHACAL equation: $(x \oplus \delta^{(j)}) - x = \Delta^{(j)}$
  - with up to m pairs $(\delta^{(j)}, \Delta^{(j)})$ known, $j = 1, \ldots, m$
  - and x fixed and unknown.

$\Rightarrow$ x can be determined by evaluating the carry flows of the addition:

$\Rightarrow$ $x_i = \delta^{(j)}_{i+1} \oplus \Delta^{(j)}_{i+1}$ if $\delta^{(j)}_i = 1$, $0 \leq i \leq 30$

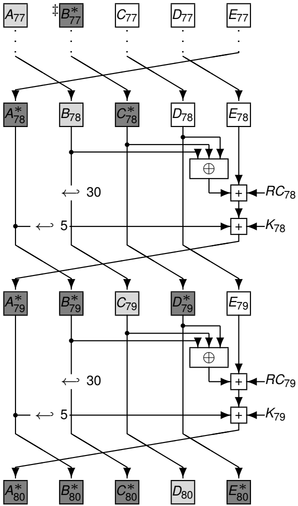
$\Rightarrow$ x can be determined (except for the highest bit) if $\bigvee_{j=1}^{m} \delta^{(j)} = (*, 1, \ldots, 1)$

DFA on SHACAL1 (FDTC 2009)

- All shaded values are known to the attacker
- Induce errors in B_{77}
- Determine D_{78} ($= E_{79}$) using the SHACAL equation:
  Taking the difference of the disturbed and undisturbed output of the $\oplus$-function yields:

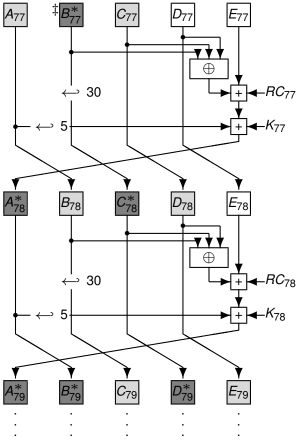
$$
\begin{aligned}
x &= BCD_{78} = B_{78} \oplus C_{78} \oplus D_{78} \\
\delta &= C^{*}_{78} \oplus C_{78} \\
\Delta &= (A^{*}_{79} - A_{79}) - ((A^{*}_{78} \hookleftarrow 5) - (A_{78} \hookleftarrow 5))
\end{aligned}
$$

- Calculate K_{79} except for the highest bit
- Strip of the last round



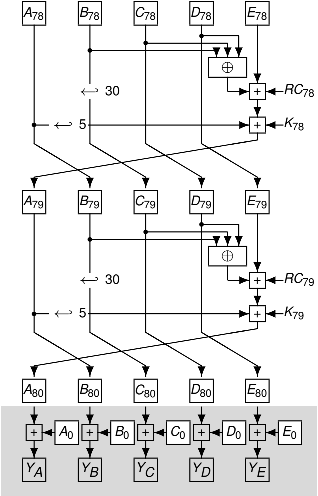
DFA on SHACAL1 (2)

- Determine D_{77} $(= E_{78})$ using the SHACAL equation
- Calculate K_{78} (except for the highest bit)
- Repeat the steps above by inducing errors in $B_{75}, \ldots, B_{63}$
- Use the key derivation function to calculate all round keys
- Validate the correct one with one plain/cipher pair



SHA1 Compression function (SHA1COMPR)

- SHA1COMPR :
  $$\{0,1\}^{160} \times \{0,1\}^{512} \longrightarrow \{0,1\}^{160},$$
  $$(P, K) \longmapsto Y$$
- $\text{SHA1COMPR}(P, K) = Y_A || Y_B || \ldots || Y_E :=$
  $$(A_{80} + A_0)||(B_{80} + B_0)|| \ldots ||(E_{80} + E_0)$$
- Only interested in the case $P = P(K_1)$ and $K = K(K_2)$ with some unknown K_1 and K_2
- The state $(A_{80}||B_{80}||C_{80}||D_{80}||E_{80})$ is not known to the attacker
- Use cases (fixed input, known output)
  - Key derivation functions
  - HMACs



DFA on SHA1COMPR - Theoretical background

- SHA1 equation:

$$\begin{aligned} Y^{(j)} - X &= \Phi^{(j)} \\ (Y^{(j)} \hookleftarrow t) - (X \hookleftarrow t) &= \Psi^{(j)}, \end{aligned}$$

  - given the rotation parameter t
  - with $\Phi^{(j)}$, $\Psi^{(j)}$ known, $Y^{(j)}$ random and unknown, $j = 1, \ldots, m$
  - and $X = X_1 \cdot 2^{32-t} + X_0$ fixed and unknown.

$\Rightarrow$ By evaluation of the borrows at the borders you can determine X_1 and X_0 with falling probability from the left to the right

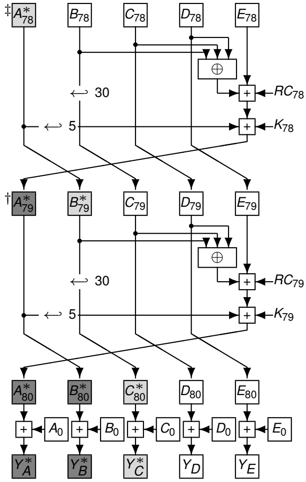
DFA on SHA1COMPR

- Elimination of the final addition
- Induce errors on A_{79}
- Build SHA1 equation:

$$
\begin{aligned}
A^*_{79} - A_{79} &= Y^*_B - Y_B \\
(A^*_{79} \hookleftarrow 5) - (A_{79} \hookleftarrow 5) &= Y^*_A - Y_A
\end{aligned}
$$

- Calculate remaining candidates for B_0
- Insert errors on A_{78}
- Build SHA1 equation to determine candidates for C_0 independent of B_0

$$
\begin{aligned}
(A^*_{78} \hookleftarrow 30) - (A_{78} \hookleftarrow 30) &= Y^*_C - Y_C \\
(A^*_{78} \hookleftarrow 5) - (A_{78} \hookleftarrow 5) &= Y^*_B - Y_B
\end{aligned}
$$



DFA on SHA1COMPR (2)

- Use SHACAL equation with

$$
\begin{aligned}
x &= BCD_{79} = B_{79} \oplus C_{79} \oplus D_{79} \\
\delta &= B_{79}^{*} \oplus B_{79} = ((Y_C^{*} - C_0) \hookleftarrow 2) \oplus ((Y_C - C_0) \hookleftarrow 2) \\
\Delta &= (Y_A^{*} - Y_A) - (((Y_B^{*} - B_0) \hookleftarrow 5) - ((Y_B - B_0) \hookleftarrow 5)),
\end{aligned}
$$

  to drop all pairs (B_0, C_0) for which the equation has no solution and compute $x = BCD_{79}$
- Notice that C_0 will be completely determined in most cases
  But: Almost no impact on the number of candidates of B_0
- Insert faults in A_{77}
- Use SHA1 equation to determine candidates for D_0
- Use SHACAL equation with $x = BCD_{78}$ to reduce the candidates of D_0

DFA on SHA1COMPR (3)

- At the end of elimination phase we know: C_0, D_0, BCD_{79} und BCD_{78}
- Calculate E_0 and E_{79} (independent of B_0):

$$\begin{aligned} E_0 &= Y_E - E_{80} = Y_E - D_{79} = \\ &\quad Y_E - (BCD_{79} \oplus (((Y_C - C_0) \hookleftarrow 2) \oplus (Y_D - D_0))) \\ E_{79} &= BCD_{78} \oplus B_{78} \oplus C_{78} = \\ &\quad BCD_{78} \oplus ((Y_D - D_0) \hookleftarrow 2) \oplus (Y_E - E_0) \end{aligned}$$

- Adapt attack of SHACAL1 to start in round 79 instead of round 80
- with $E^*_{79} = E_{79} + Y^*_A - Y_A +$
  $((Y_B - B_0) \hookleftarrow 5) - ((Y^*_B - B_0) \hookleftarrow 5) +$
  $(((Y_C - C_0) \hookleftarrow 2) \oplus (Y_D - D_0) \oplus (Y_E - E_0)) -$
  $(((Y^*_C - C_0) \hookleftarrow 2) \oplus (Y^*_D - D_0) \oplus (Y^*_E - E_0))$

Computational optimization (Impact of B_0)

- B_0 influences
  - A_{79} and A^*_{79}, but not $A_{79} - A^*_{79}$
  - K_{78}, but $K_{78} + B_0 = const$ for all B_0
  - $E^*_{79} = E_{79} + Y^*_A - Y_A + ((Y_B - B_0) \hookleftarrow 5) - ((Y^*_B - B_0) \hookleftarrow 5) + \ldots$
- Idea: If the 5 upmost bits of B_0 are known,
  - choose $\hat{X} \in \{\min \text{of all } B_0's, \max \text{of all } B_0's\}$ and
  - discard all errors Y^*_B equal to one of the remaining candidates of B_0, then

$$((Y_B - B_0) \hookleftarrow 5) - ((Y^*_B - B_0) \hookleftarrow 5) = ((Y_B - \hat{X}) \hookleftarrow 5) - ((Y^*_B - \hat{X}) \hookleftarrow 5)$$

$\Rightarrow$ E^*_{79} does not "really" depent on B_0

$\Rightarrow$ Only two values of B_0 have to be tested

Fault model

- Used a 32 bit faultmodel
- Also 16bit (and 8bit) fault model simulated
  - Needed less errors to determine B_0, C_0 and D_0 with the SHA1 equation
  - Possibility to attack the final addition directly
  - But up to a factor of 2(4) more errors for the SHACAL equation
  - Costs less computation time (not critical)

Simuation results (32 bit faultmodel)

Nr. of faults (elimination-phase)	remaining candidates (avg)					Nr. of faults (SHACAL1-phase)	success rate	computation time (avg)	Nr. of simulations
	B_0	C_0	D_0	E_0	E_{79}				
$3 \cdot 1622 = 4866$	162406	1.8	1.7	7.7	22.3	$9 \cdot 15 = 135$	93.8%	3.9 min	500
$3 \cdot 1288 = 3864$	215834	1.6	1.9	6.8	25.4	$9 \cdot 15 = 135$	93.0%	6.6 min	500
$3 \cdot 955 = 2865$	291700	1.7	1.6	6.2	16.0	$9 \cdot 15 = 135$	89.2%	7.5 min	500
$3 \cdot 622 = 1866$	450078	1.8	1.7	6.3	18.0	$9 \cdot 15 = 135$	88.0%	25 min	500
$3 \cdot 455 = 1365$	565252	1.6	1.7	6.1	19.4	$9 \cdot 15 = 135$	76.5%	82 min	153
$3 \cdot 289 = 867$	954745	1.6	1.5	7.7	22.4	$9 \cdot 15 = 135$	73.7%	171 min	114

- Impact of number of faults only to remaining candidates of B_0, the success rate and the computation time
- Memory overflows (to many candidates of B_0) or timeout restrictions lowered success rate
- Change program to dynamic fault injection to get results independent of the success rate

Conclusion and ongoing work

- With about 1000 faults it is possible to fully extract the secret inputs of the SHA1 compression function with high probability
- Work on SHA224/256 similar to the work done by Wei Yue-chuan et al. on SHACAL-2

  (Wei Yue-chuan, Li Lin, Li Rui-lin and Li Chao, Differential Fault Analysis on SHACAL-2, Journal of Electronics and Information Technology, 2010)

The End

- Thank you for your attention