Algebraic Fault Analysis on GOST for Key Recovery and Reverse Engineering

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Fault Diagnosis and Tolerance in Cryptography

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Outline

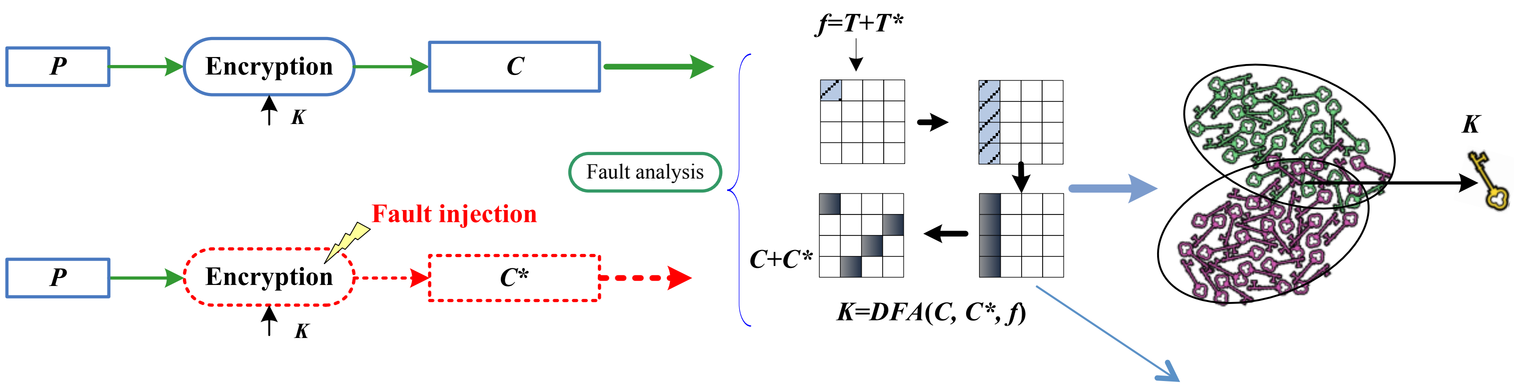
- **Motivation?** Algebraic Fault Analysis
- **Target?** GOST and Attack Scenarios
- **Technique?** AFA on GOST
- **Results?** Key Recovery and Reverse Engineering
- **Summary?** Conclusion of Our Work

Traditional Fault Analysis

FA (Fault Attack) first proposed by Boneh et al in 1996.

– Received faulty output, guess the fault, find the secret.

- DFA (Differential Fault Analysis) proposed by Biham and Shamir in 1997.
  – Used to break public-key ciphers (ECC), block ciphers (AES, ARIA, Camellia and CLEFIA) and stream ciphers (RC4, Trivium).

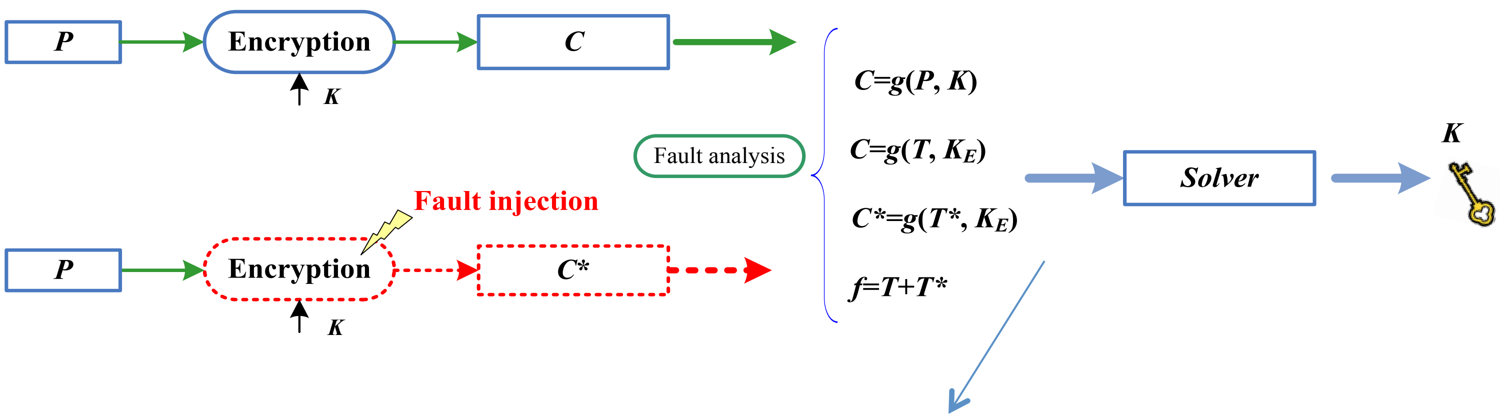


Framework of DFA

Manually fault analysis;
Maximal efficiency unknown？

Algebraic Fault Analysis

- AFA (Algebraic Fault Analysis) proposed by Courtois in 2010.
  - Algebraic cryptanalysis with fault attack.



Compared with DFA:

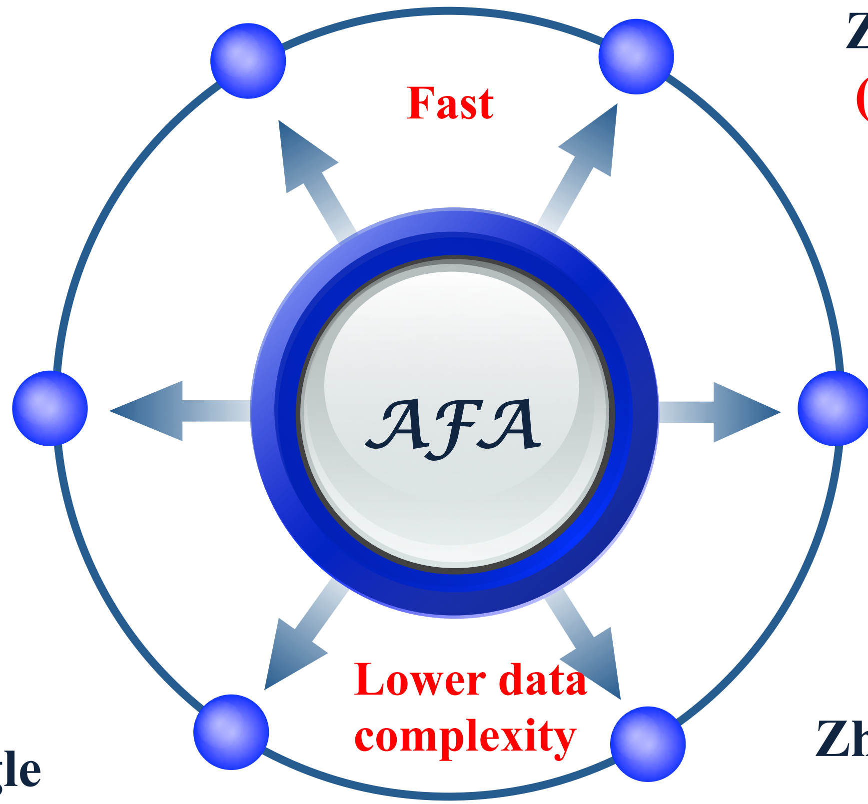
- Algebraic analysis are generic and automatic
- Solvers (automatic) allow easier and simpler analysis
- Fault information allows optimization

State-of-the-art AFA

eSmart 2010
Courtois: DES, single fault, $2^{17.35}$ hours

COSADE 2011
Mohamed: Trivium, less faults

ePrint 2012/400
Jovanovic: LED, single fault, 14.67 hours.



COSADE 2013
Zhang: Piccolo, DES (10 seconds), MIBS, single fault

FDTC 2013
Zhao: LED, single fault, 1-3 minutes, evaluating DFA

CACR 2013
Zhao: LBlock, single fault

Our Motivations?

- Current AFA
  - Key recovery when the design of cipher is known
  - Evaluating the reduced key search space of DFA
- Our work
  - Can AFA work when partial design of cipher is unknown?
  - Can AFA be used for reverse engineering besides key recovery?

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Overview of GOST

- A Soviet and Russian government standard symmetric key block cipher.
  - 64-bit block cipher
  - 256 bit keys
  - 32 rounds
  - Feistel structure
  - 8 S-Boxes
  - modulo 2^{32} nonlinear part
  - Simple key schedule

Overview of GOST

- processes the right half of the block using function f, XORs the result from f with the left half, and swaps the two halves.
- key schedule is simple, divide 256-bit key into 8 pieces, using one piece per round

the contents of 8 S-Boxes

might be secret

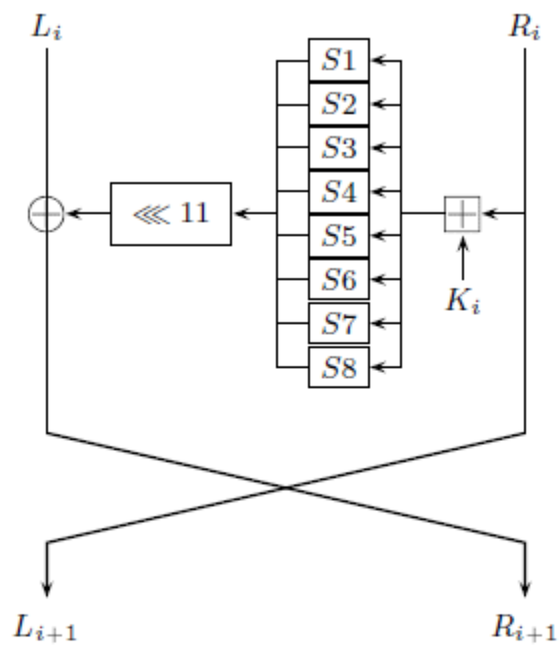

Figure 1. One round of GOST

Attack Scenarios

single byte fault injection on the right half of GOST

- **Scenario 1:** known complete GOST design, key recovery?

- **Scenario 2:** 8 S-Boxes secret, known secret key, AFA technique, reverse engineering of S-Boxes?

- **Scenario 3:** 8 S-Boxes secret, unknown secret key, AFA technique, both key recovery and reverse engineering?

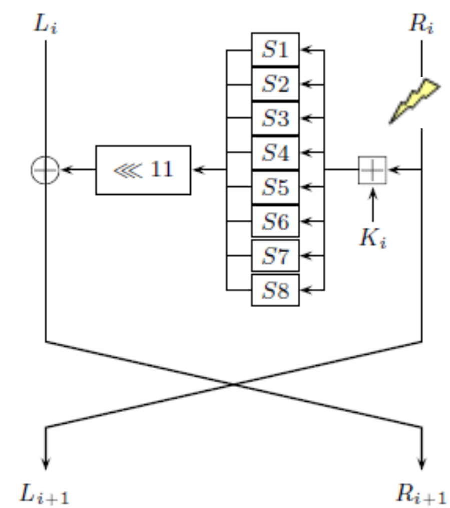

Figure 1. One round of GOST

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AFA on GOST

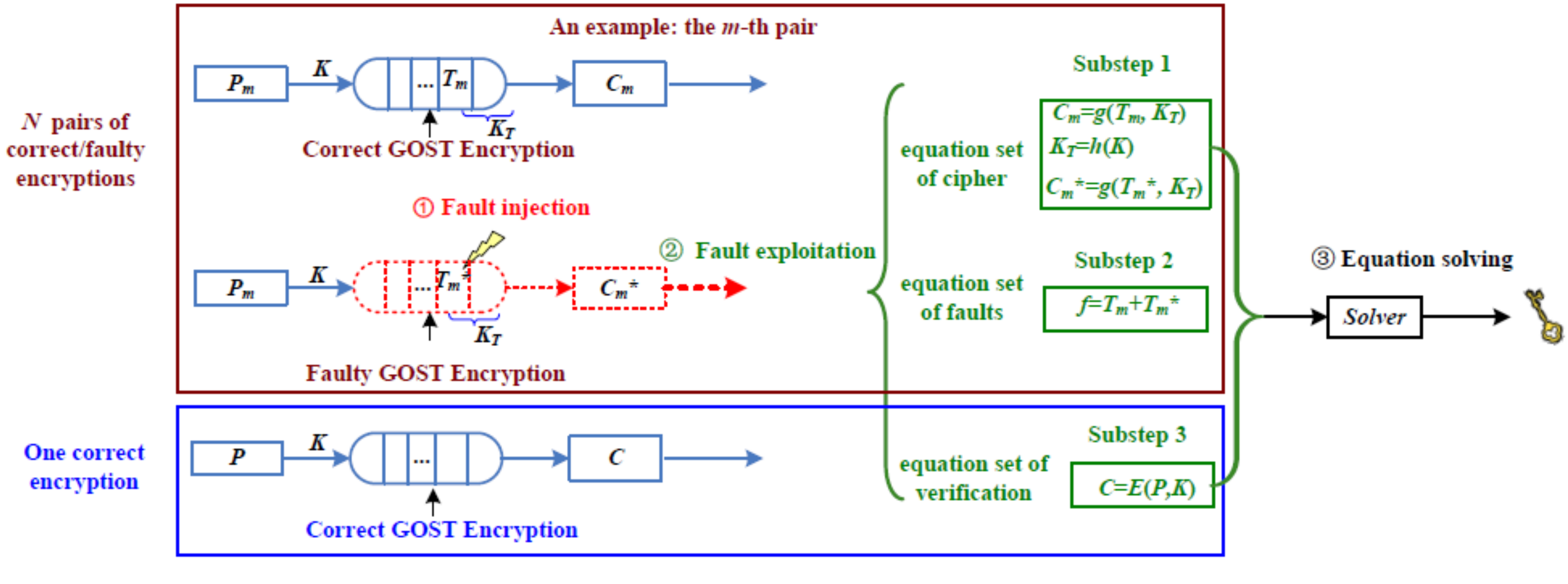

Figure 2. Framework of AFA on GOST

- one full correct GOST equation set
- the last few GOST rounds equation set since the fault injections for N pairs of correct and faulty encryptions

Step 1: GOST Equation Set

- Represent AK (Adding modulo 2^{32})

$$
\begin{aligned}
&z_{32} = x_{32} + y_{32} \\
&t_{31} = x_{32}y_{32} \\
&z_{31} = x_{31} + y_{31} + t_{31} \\
&t_{30} = x_{31}y_{31} + x_{31}t_{31} + y_{31}t_{31} \\
&z_{30} = x_{30} + y_{30} + t_{30} \\
&t_{29} = x_{30}y_{30} + x_{30}t_{30} + y_{30}t_{30} \\
&z_{29} = x_{29} + y_{29} + t_{29} \\
&t_{28} = x_{29}y_{29} + x_{29}t_{29} + y_{29}t_{29} \\
&\dots \\
&z_2 = x_2 + y_2 + t_2 \\
&t_1 = x_2y_2 + x_2t_2 + y_2t_2 \\
&z_1 = x_1 + y_1 + t_1
\end{aligned}
$$

Step 1: GOST Equation Set

- Represent SL (S-Box lookup)

$$
\begin{aligned}
y_1 &= x_2 + x_3 + x_4 + x_1x_2 + x_1x_3 + \\
&\quad x_2x_4 + x_1x_2x_4 + x_2x_3x_4 \\
y_2 &= 1 + x_3 + x_4 + x_3x_4 + x_1x_2x_3 + \\
&\quad x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 \\
y_3 &= x_1 + x_4 + x_1x_3 + x_1x_4 + x_2x_4 + \\
&\quad x_1x_2x_4 + x_2x_3x_4 \\
y_4 &= x_2 + x_3 + x_1x_4 + x_2x_4 + x_3x_4 + \\
&\quad x_1x_2x_3 + x_1x_3x_4
\end{aligned}
$$

Public S-Box

$$
\begin{aligned}
y_1 = a_1 &+ a_2x_1 + a_3x_2 + a_4x_3 + a_5x_4 + \\
&a_6x_1x_2 + a_7x_1x_3 + a_8x_1x_4 + \\
&a_9x_2x_3 + a_{10}x_2x_4 + a_{11}x_3x_4 + \\
&a_{12}x_1x_2x_3 + a_{13}x_1x_2x_4 + a_{14}x_1x_3x_4 + \\
&a_{15}x_2x_3x_4 + a_{16}x_1x_2x_3x_4 \\
y_2 = a_{17} &+ a_{18}x_1 + a_{19}x_2 + a_{20}x_3 + a_{21}x_4 + \\
&a_{22}x_1x_2 + a_{23}x_1x_3 + a_{24}x_1x_4 + \\
&a_{25}x_2x_3 + a_{26}x_2x_4 + a_{27}x_3x_4 + \\
&a_{28}x_1x_2x_3 + a_{29}x_1x_2x_4 + a_{30}x_1x_3x_4 + \\
&a_{31}x_2x_3x_4 + a_{32}x_1x_2x_3x_4 \\
y_3 = a_{33} &+ a_{34}x_1 + a_{35}x_2 + a_{36}x_3 + a_{37}x_4 + \\
&a_{38}x_1x_2 + a_{39}x_1x_3 + a_{40}x_1x_4 + \\
&a_{41}x_2x_3 + a_{42}x_2x_4 + a_{43}x_3x_4 + \\
&a_{44}x_1x_2x_3 + a_{45}x_1x_2x_4 + a_{46}x_1x_3x_4 + \\
&a_{47}x_2x_3x_4 + a_{48}x_1x_2x_3x_4 \\
y_4 = a_{49} &+ a_{50}x_1 + a_{51}x_2 + a_{52}x_3 + a_{53}x_4 + \\
&a_{54}x_1x_2 + a_{55}x_1x_3 + a_{56}x_1x_4 + \\
&a_{57}x_2x_3 + a_{58}x_2x_4 + a_{59}x_3x_4 + \\
&a_{60}x_1x_2x_3 + a_{61}x_1x_2x_4 + a_{62}x_1x_3x_4 + \\
&a_{63}x_2x_3x_4 + a_{64}x_1x_2x_3x_4
\end{aligned}
$$

Secret S-Box

64 variables a_i are introduced

Step 1: GOST Equation Set

- Represent RL (Rotating bits to left)

$$y_i = x_{((i+9) \ mod \ 32)+1}$$

- Represent GOST decryption can accelerate speed of AFA)

Algorithm 1. Building the equation set for r rounds decryption of GOST

```
C ⟵ [c_1, c_2, ..., c_64]
L_33 ⟵ [c_1, c_2, ..., c_32]
R_33 ⟵ [c_33, c_34, ..., c_64]
    L_32 ⟵ L_33 ⊕ RL(SL(RL(R_33, K_32)))
    R_32 ⟵ R_33
for i =31 to 32 − r (i > 0) do
    L_i ⟵ R_{i+1}
    R_i ⟵ L_{i+1} ⊕ RL(SL(RL(R_{i+1}, K_i)))
end for
```

Step 2: Fault Equation Set

- Suppose Z denote the injected fault difference
  - Z can be considered as the concatenation of four bytes

$$Z_1||Z_2||Z_3||Z_4, \quad Z_i = (z_{8i-7}, z_{8i-6}, \ldots, z_{8i})\ (1 \leq i \leq 4).$$

  - Four one-bit u_i are used to represent whether Z_i is faulty (u_i=0) or not

$$u_i = (1 \oplus z_{8i-7}) \wedge (1 \oplus z_{8i-6}) \wedge (\ldots) \wedge (1 \oplus z_{8i})$$

  - Only one byte fault is injected, only one u_i=0

$$(1 - u_1) \vee (1 - u_2) \vee (1 - u_3) \vee (1 - u_4) = 1,$$
$$u_i \vee u_j = 1, \quad 1 \leq i < j \leq 4$$

Step 3: Solver

- Combine the equation set of GOST with injected fault and use solver to recover the secret key.
- CryptoMiniSAT v2.9.4, support multiple solution output
- The PC that runs CryptoMiniSAT has the following configuration: Intel Core I7-2640M, 2.80 GHZ, and 4G bytes memory. The operating system is 64-bit Windows 7.

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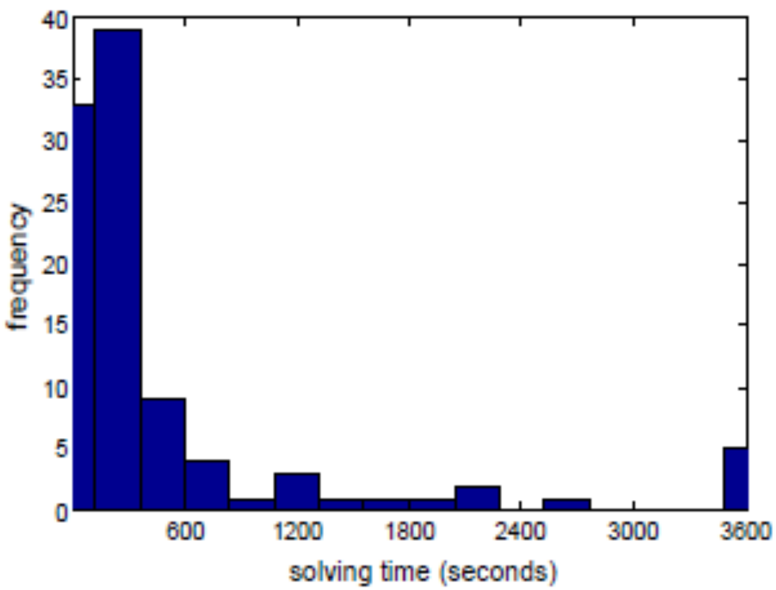
Experiment Parameters

N	**the number of fault injections**
$V(N)$	the number of variables in equation set
$A(N)$	the number of ANF equations in equation set
$\upsilon(N)$	the size of the generated scripts
$t(N)$	the time complexity (seconds) required in solver
τ	threshold of the time complexity (seconds) in a successful AFA
$\varphi(N,\tau)$	the success rate
$\lambda(N)$	the entropy of the secret key in Scenario 1

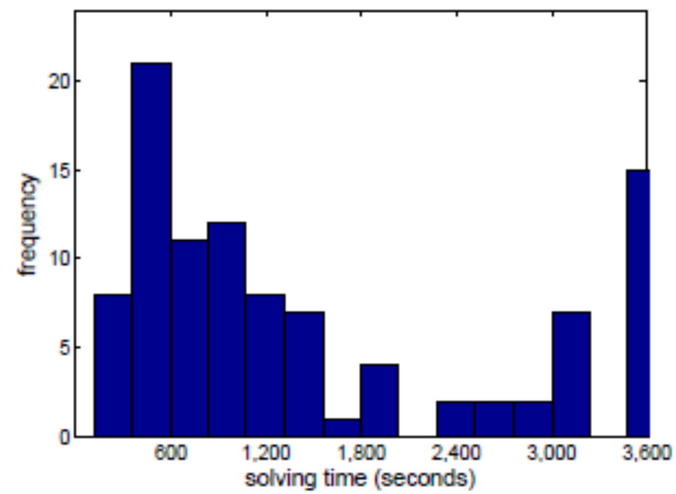
Results of Scenario 1

$4n$ random faults are injected into R_i, i ={24, 26, 28, 30} of GOST (n faults for each i, $N = 4n$).



(a) N=12, $V(N) = 42,857$, $A(N) = 83,037$, $v(N) =$ 1613KB

$\lambda(N)=2^{12.2}$



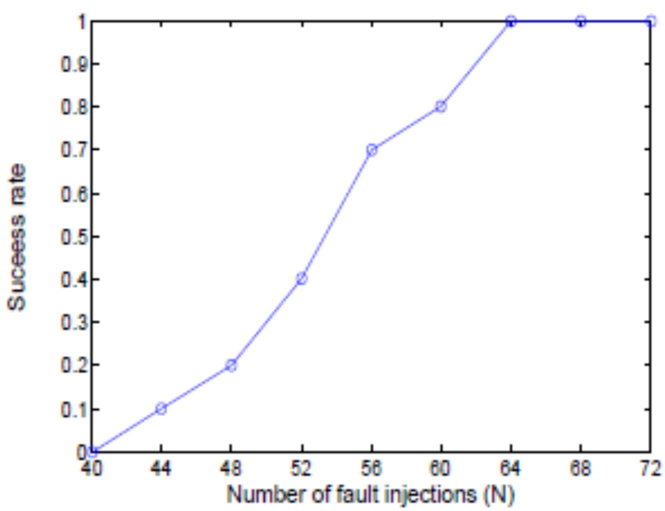
(b) N=8, $V(N) = 32,913$, $A(N) = 63,689$, $v(N) =$ 1226KB

$\lambda(N)=2^{16.7}$

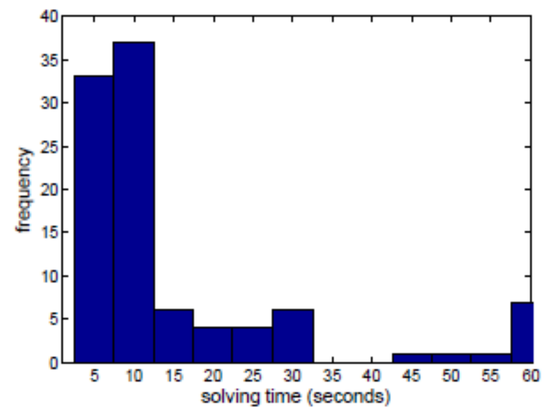
N=8 faults are required to recover the master key, which is less than 64 in [Kim10].

Results of Scenario 2

2*n* random faults are injected into R_i, i ={30, 31} of GOST (*n* faults for each *i*, $N = 2n$).



(a) Success rate (different N)



(b) Solving time (N=64, $V(N) = 183,553$, $A(N) = 467,969$, $v(N) = 9024$KB)

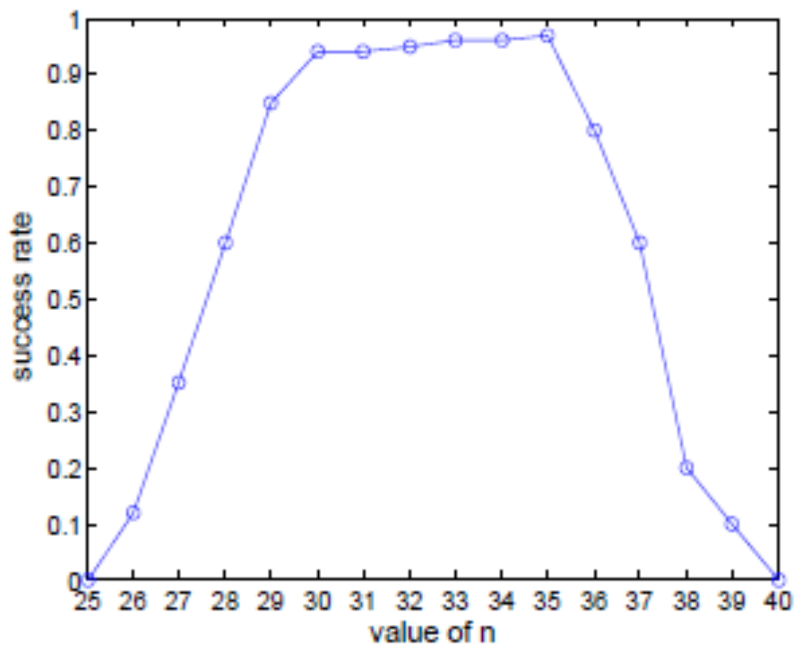
64 faults to recover the 8 S-Boxes

64-BIT SECRET PARAMETERS FOR THE EIGHT RECOVERED S-BOXES OF GOST (IN HEXADECIMAL)

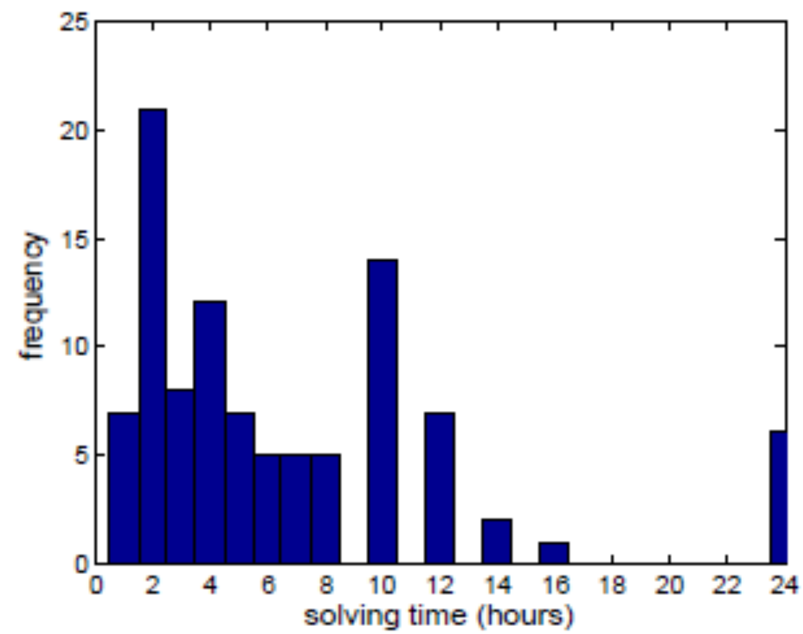
S-box	$a_1, a_2, \ldots, a_{64}$	S-box	$a_1, a_2, \ldots, a_{64}$
$S1$	0x3e4a983e4b4a3174	$S5$	0x0ab8873cec12349e
$S2$	0xf478c97494c208a6	$S6$	0x1dceda3679486e34
$S3$	0x6986bf52669eec3c	$S7$	0xf5c8eb982aead2b2
$S4$	0x5802b282ac52f22e	$S8$	0x5c4a3b560eba85b6

Results of Scenario 3

9*n* random faults are injected into R_i, *i* ={23,24,25,26,27, 28,29,30,31} of GOST (*n* faults for each *i*, *N* = 9*n*).



(a) Success rate $\phi(N,\tau)$, $N = 9n$



(b) Solving time (N=270, $V(N) = 1,543,797$, $A(N) = 3,974,183$, $v(N) = 83,700$KB)

270 faults for the recovery of both of the key and 8 S-Boxes

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Conclusion of Our Work

Make a comprehensive study of AFA on GOST

- **AFA is Efficient**: when the whole design of GOST is known, the key recovery requires only 8 fault injection, less than 64 in previous DFA work.
- **AFA is Powerful:** can be used for reverse engineering, even both the key and S-Boxes are secret.
- **AFA is Automatic**: no need to analyze the fault propagation.
- **AFA is Generic**: apply to different attack scenarios.
- **One lesson:** keeping some components in a cipher secret cannot guarantee its security.

Thanks!

Q & A

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