A Practical Second-Order Fault Attack against a Real-World Pairing Implementation

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joint work with

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Jean-Pierre Seifert

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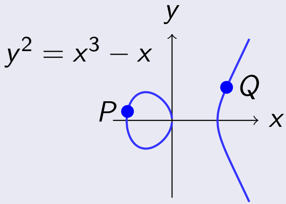
The role of the final exponentiation

Two step pairing computation

$$
\begin{array}{rcl}
e : \mathcal{E}(\mathbb{F}_{p^k}) \times \mathcal{E}(\mathbb{F}_{p^k}) & \to & \mathbb{F}^*_{p^k} \\
(P, Q) & \mapsto & f_{r,P}(Q)^d
\end{array}
$$



1. $\alpha \leftarrow f_{r,P}(Q)$ (Miller step)
2. $\beta \leftarrow \alpha^d$ with $d = (p^k - 1)/r$ (final exponentiation)

1. Computes non-degenerate, bilinear mapping to $\mathbb{F}^*_{p^k}/(\mathbb{F}^*_{p^k})^r$.
2. Maps equivalence classes $\mathbb{F}^*_{p^k}/(\mathbb{F}^*_{p^k})^r$ to unique representatives in μ_r.

Fault attacks on pairings

Cryptanalysis of pairings

Inversion of both steps is required

1. Search secret Q as one solution of $f_{r,P}(x,y) = \alpha$
   Problem: Function $f_{r,P}(x,y)$ has huge degree r
2. Inversion of final exponentiation $(\,\cdot\,)^d = \beta$
   Problem: Difficult to identify the correct d-th root α

$\Rightarrow$ 2nd order attacks required

Cryptanalysis of pairings with fault attacks

1. Reduce degree of $f_{r,P}(x,y)$ by modification of r, P, Q
2. Make $(\,\cdot\,)^d$ as injective as possible

Fault attacks on pairings

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$\Rightarrow$ 2nd order attacks required

Our 2nd order attack

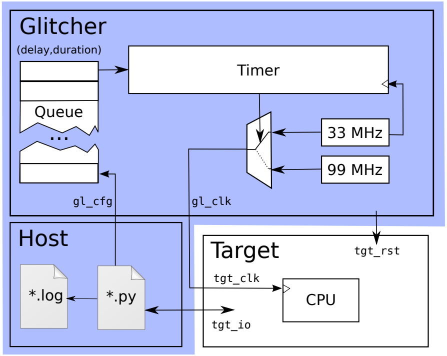
1. Round reduction of Miller loop: obtain $f_{r',P}(x,y)$ of degree $r' = 5$.
2. Skipping final exponentiation

Fault attacks on pairings

Outline of our attack

1. Assumption: attacker with physical access to target (especially CPU clock)
2. Trigger computation of $e(P, Q)$ on public argument (e.g. P) and secret argument (e.g. Q)
3. Distort computation of $e(P, Q)$ by clock glitch to obtain β'
4. Compute secret Q from β'

Our attack: Schematic Hardware Setup



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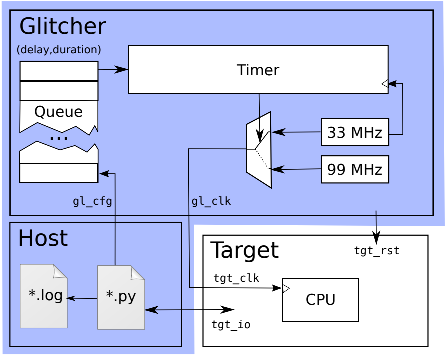
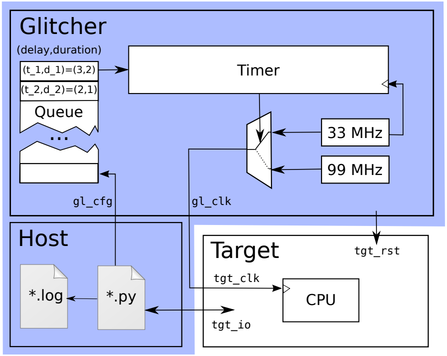


- Mechanism: CPU clock glitching
- Effect: Instruction skips



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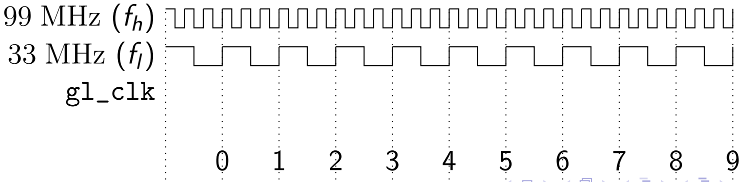

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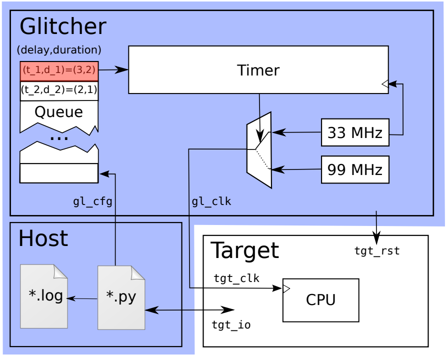

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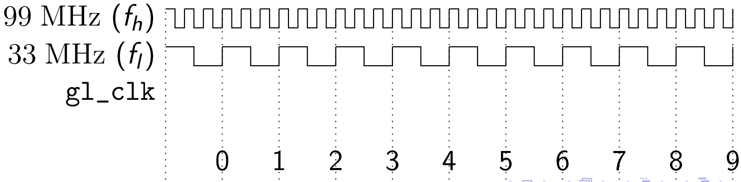

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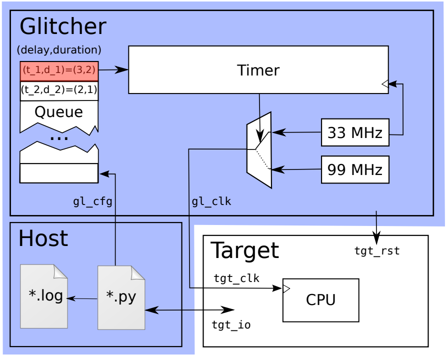

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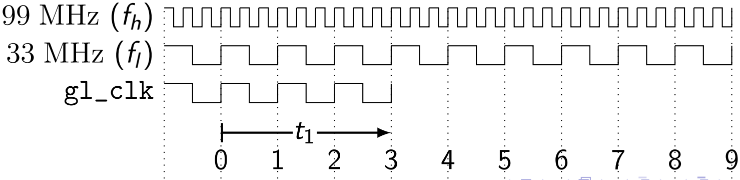

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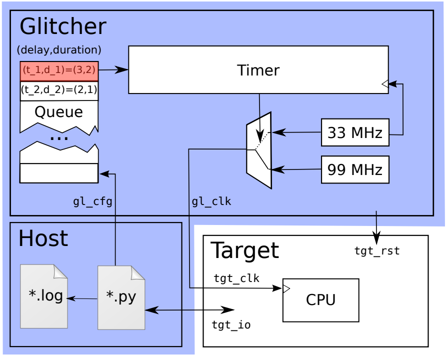

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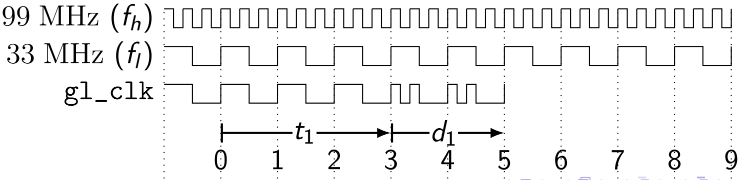

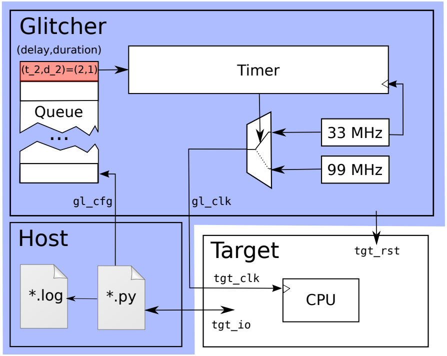

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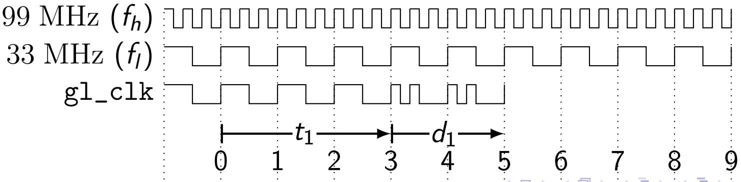

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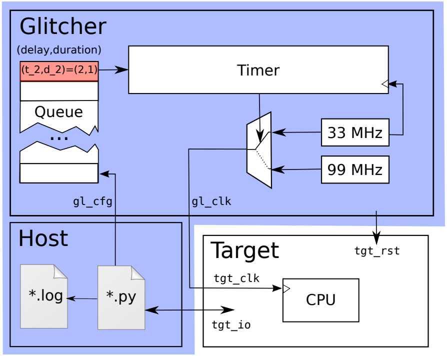

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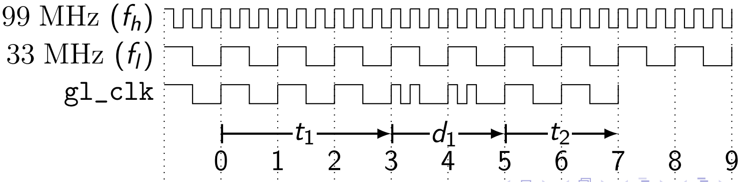

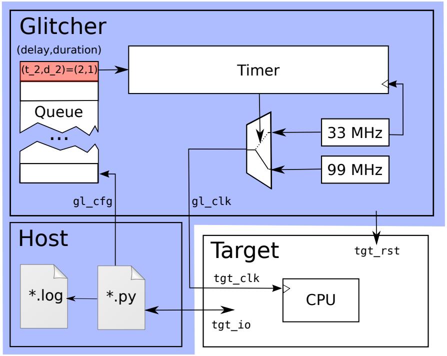

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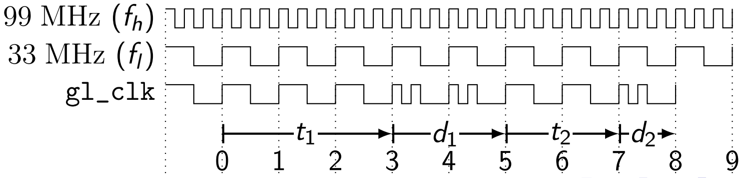

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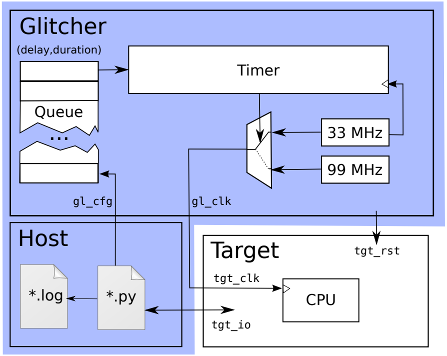

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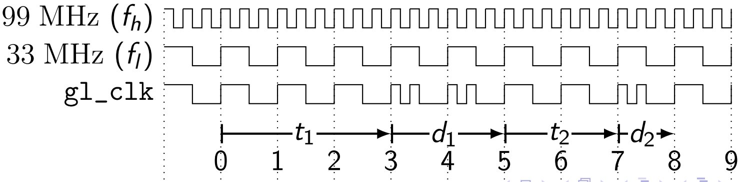
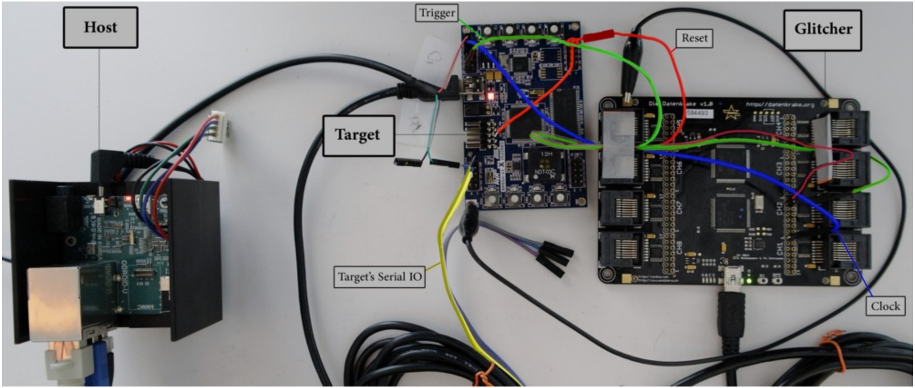
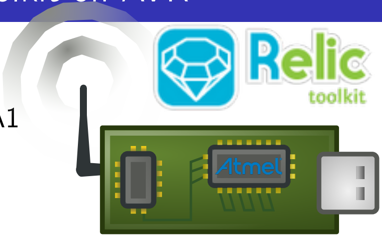


Our attack: Real Hardware Setup



Our target: Eta pairing of Relic toolkit on AVR



- Target hardware: Atmel AVR Xmega A1
- Target Software: Relic toolkit
  - Open source
  - Prime and Binary field arithmetic
  - Elliptic curves over prime and binary fields (NIST curves and pairing-friendly curves)
  - Bilinear maps and related extension fields
  - Cryptographic protocols
- Combination used on wireless sensor nodes as TinyPBC
- Unmodified code
  - No additional NOPs
  - No monitors
  - No triggers

The RELIC implementation

Input $P, Q \in \mathcal{E},\ r = (r_n \dots r_0)$
Output $f_{r,P}(Q)$

$T \leftarrow [2]P$
$\alpha \leftarrow l_{P,P}(Q) \cdot l_{(r-1)P,P}(Q)$
for $j \leftarrow n-2, \dots, 1$ **do**
  if $r_j = 1$ **then**
    $T \leftarrow T + P$
    $\alpha \leftarrow \alpha \cdot l_{T,P}(Q)$
  end if
  $T \leftarrow [2]T$
  $\alpha \leftarrow \alpha^2 \cdot l_{T,T}(Q)$
end for
$\alpha \leftarrow \alpha^d$
return α

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10: **end for**
11: $\alpha \leftarrow \alpha^d$
12: **return** α

```
.L2
...
call fb4_mul_dxs
subi r16,1
sbc r17,__zero_reg__
breq .+2
rjmp .L2
subi r28,36
...
movw r22,r28
movw r24,r28
call etat_exp
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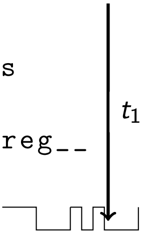

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t_1

t_2

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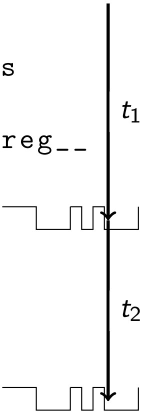

The RELIC implementation

Relic toolkit

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t_1

t_2

Host: Outline of Analysis

1. Output with successful glitch:

$$\beta' = \left(l_{P,P}(Q) \cdot l_{(n-1)P,P}(Q)\right)^2 \cdot l_{2P,2P}(Q)$$

2. Capture secret as root of polynomial of degree 5:

$$f(x,y) \quad = \quad \beta' \; - \; \left(l_{P,P}(x,y) \cdot l_{(r-1)P,P}(x,y)\right)^2 \cdot l_{2P,2P}(x,y)$$

3. Compute simultaneous roots of $f(x,y)$ and $\mathcal{E} : y^2 = x^3 - x$
4. Test candidates Q' against result of correct pairing:

$$e(P,Q') = e(P,Q)?$$

Timing of first fault is critical

The challenge for 2nd order attack

- The timings t_1, t_2 of the target instructions depend on unknown secret (e.g. Q)
- It is not possible to detect case where only one glitch is successful
- Many combinations have to be tested

Our strategy

1. Profiling: Determine probability distribution of t_1 and t_2 for randomized secret input (e.g. Q)
2. Attack:
   - Rank candidates for t_1 and t_2 according to their probability
   - Introduce fault as early as possible
3. Analysis: Full Automation

Host: Outline of Analysis

1. Output with successful glitch:

$$\beta' = \left(l_{P,P}(Q) \ \ldots \ )\right)^2 \cdot l_{2P,2P}(Q)$$

2. Capture secret as roo[illegible] 5:

$$f(x,y) \quad \ldots \quad {}_{,P}(x,y)\big)^2 \cdot l_{2P,2P}(x,y)$$

3. Compute simult[illegible]) and $\mathcal{E} : y^2 = x^3 - x$
4. Test candidates Q [illegible] of correct pairing:

$$e(P, Q') = e(P, Q)?$$

Automation with sage

Performance of the attack

- In $<$ 10 seconds per experiment (average):
  - Self-tests
  - Configure glitcher
  - Restart target
  - Induce faults
  - Analyze result
- More than 10000 experiments per day

Conclusion

- Second order attacks on pairings possible
- Two stage computation: not enough protection
- Add dedicated countermeasures as protection against active attacks

Simplifiy inversion of final exponentiation with faults

Ongoing work

Problem

Final exponentiation cannot always be skipped:

- Inlining at higher optimization levels
- Countermeasures that guarantee execution of function

Example (Inlining at higher optimization levels)

```
...
movw r22,r28
movw r24,r28
call etat_exp
pop r29
...
```

```
...
movw r22,r28
movw r24,r28
jmp etat_exp
nop
...
```

Simplifiy inversion of final exponentiation with faults

Ongoing work

Our approach

- Skip part of final exponentiation to modify exponent:

$$d = (p^k - 1)/r \quad \rightarrow \quad d' = (p^k - 1)/r + \delta$$

- Simplify mathematical inversion of final exponent d'

References

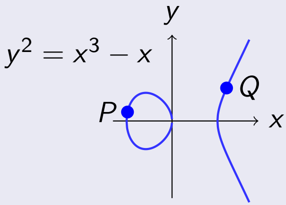
- Relic toolkit:
  http://code.google.com/p/relic-toolkit/
- Glitcher Die Datenkrake:
  https://www.usenix.org/conference/woot13/workshop-program/presentation/nedospasov

Background

The basic building block

Bilinear mapping:

$$\begin{aligned} \mathrm{e} : \mathcal{E}(\mathbb{F}_{p^k}) \times \mathcal{E}(\mathbb{F}_{p^k}) &\rightarrow \mathbb{F}^*_{p^k} \\ (P, Q) &\mapsto f_{n,P}(Q)^d \end{aligned}$$



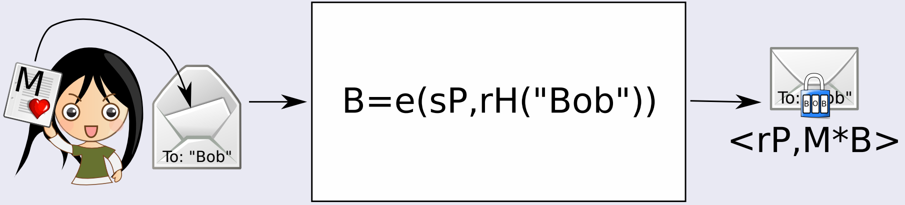
- n, d are huge
- $f_{n,P}(x, y)$: zero of order n at P, degree $> n$

Interesting properties for application in cryptography

- Bilinearity: $e(aP, bQ) = e(P, Q)^{ab} = e(bP, aQ)$
- Hard to invert
- $f_{n,P}(Q)$ is efficiently computable with Miller algorithm

An example Application: IBE

Encryption



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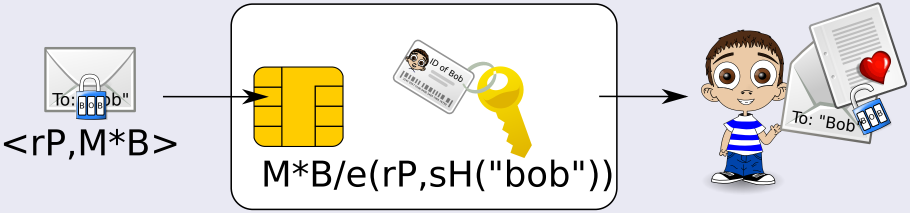

An example Application: IBE

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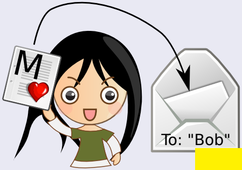


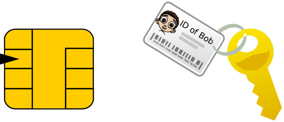
B=e(sP,rH("Bob"))

<rP,M*B>

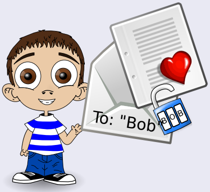
The secret decryption key is one argument of the pairing.

Decryption

<rP,M*B>



M*B/e(rP,sH("bob"))



Miller Algorithm (Victor Miller 1986)

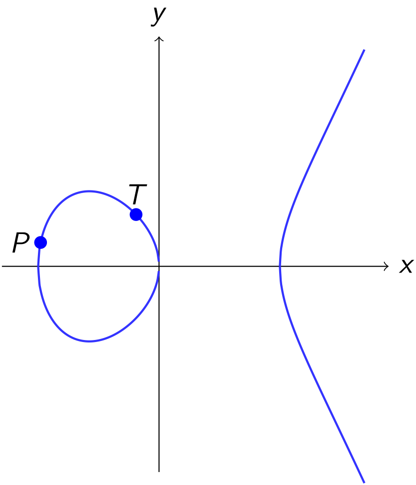
Extending the elliptic curve double and add algorithm

Input $P, Q \in \mathcal{E}$, $n = (n_{t-1} \dots n_0)$

Output $f_{n,P}(Q)$

```
α ← 1, T ← P
for j ← t − 2, ..., 0 do
    α ← α² · l_{T,T}(Q)/v_{2T}(Q)
    T ← 2T
    if n_j = 1 then
        α ← α · l_{T,P}(Q)/v_{T+P}(Q)
        T ← T + P
    end if
end for
return α^d
```

$y^2 = x^3 - x$



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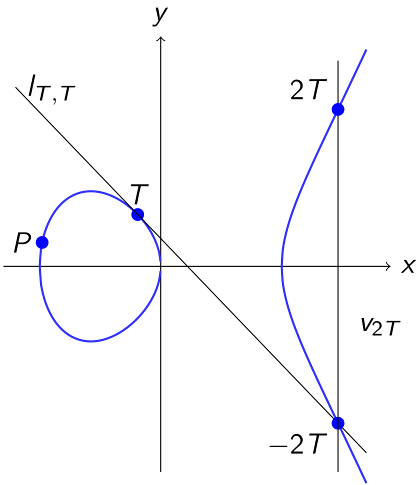

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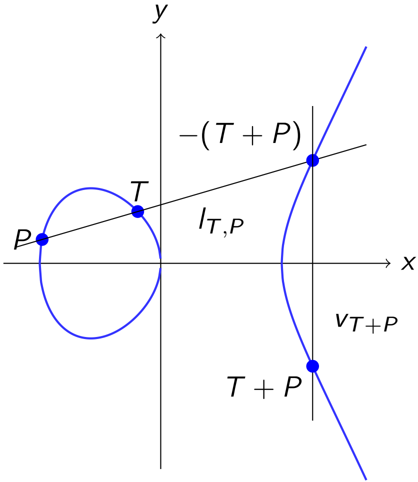

Different delays/instructions, same effect

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subi  r16,1                //re-sets zero flag: Z=0
sbc  r17,__zero_reg__      //Z=Z
breq  .+2                  //Z=1: branch
rjmp  .L2
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...
call  fb4_mul_dxs          //sets zero flag: Z=1
subi  r16,1                //re-sets zero flag: Z=0
sbc  r17,__zero_reg__      //Z=Z
breq  .+2                  //Z=1: branch
rjmp  .L2
subi  r28,36
...
```

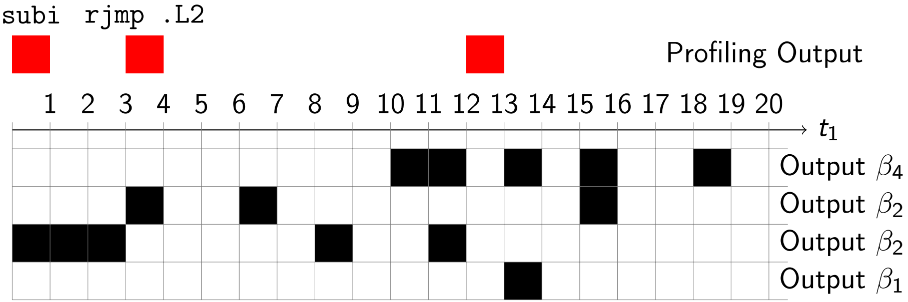
Different delays/instructions, same effect

```
.L2
...
call fb4_mul_dxs          //sets zero flag: Z=1

sbc r17,__zero_reg__      //Z=Z
breq .+2                  //Z=1: branch
rjmp .L2
subi r28,36
...
```

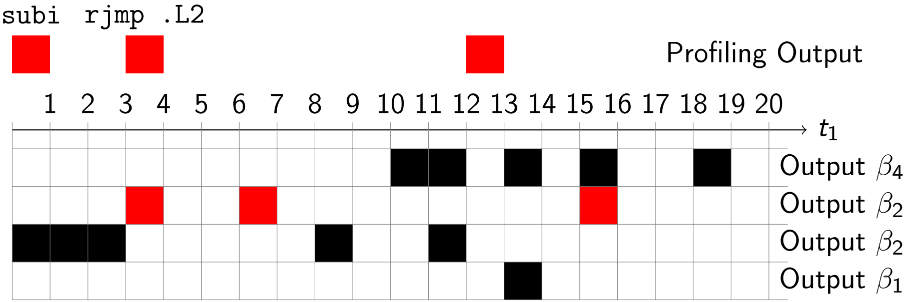
1st order attack

Group delays by output and locate `rjmp .L2`



1st order attack

Group delays by output and locate `rjmp .L2`



- Output β_2 matches pattern of profiling
- $\Rightarrow t_1 = 6$ is instruction of `rjmp .L2`
- Proceed with 2nd order attack and correct setting of t_1